5 Further reading

I give some recommendations for where to learn more about the topics touched upon in these lectures. The list is by no means complete. If you are interested in the history of birdtracks, I recommend Sec. 4.9 of THE BOOK.

General introductions to and compendiums for birdtracks

- THE BOOK on birdtracks is Predrag Cvitanović's *Group Theory: Birdtracks, Lie's and Exceptional Groups* [1]. You want to have it next to you whenever you do birdtrack calculations. A precursor are Cvitanović's 1984 lecture notes [8] which already contain a lot of the material covered in THE BOOK. The 1976 paper [9] provides a nice introduction and summary of birdtracks for SU(N) and QCD.
- Roger Penrose developed birdtracks to be used for tensors in general relativity. You get a nice impression from his semi pop-science book *The Road to Reality* [2]. There, birdtrack diagrams are introduced in Secs. §12.8, §13.3–§13.9, §14.3, §14.4, §14.6, and §14.7 in the context of differential geometry and later (Secs. §19.2, §19.6, §22.12, §26.2, §29.5) used for general relativity and other topics.
- The book *Diagram Techniques in Group Theory* by Geoffrey E. Stedman [10] also treats many aspects and applications of the the birdtrack method; it also contains an introductory section on vector algebra.
- Yuri L. Dokshitzer's lecture notes *Perturbative QCD (and beyond)* [11] introduce birdtrack techniques using QCD processes as examples; some of his notational conventions differ slightly differ from ours.
- If you are looking for a cheat sheet on birdtracks I suggest App. A of my paper [6] with Malin Sjödahl.

Background

Classic results on the representation theory of finite groups (such as S_n) and compact Lie groups (such as SU(N)), on Schur's lemma, or on the multiplication of Young diagrams can, e.g., be found in [12–14] and in a vast number of other textbooks.

Details on the specific topics (of later sections) of this course

- Young operators (non-Hermitian) in birdtracks are discussed in [15].
- The main references for the construction of multiplet bases are [3, 6]. Hermitian Young operators are constructed in [3], gluon projectors and the general rules for constructing multiplet bases are derived in [6].

  Hermitian Young operators and multiplet basis for few quarks appear already in [1, 16, 17]. The multiplet basis for $A^{\otimes 2} \to A^{\otimes 2}$ in birdtracks is constructed in [1] using a different method.

  Simplification rules for a more efficient construction of Hermitian Young operators are derived in [4, 5]. The corresponding quark multiplet bases are discussed in [18]
- When pen and paper calculations become unwieldy, the Mathematica and C++ packages [19, 20] by Malin Sjödahl come in handy.
- Sjödahl and co-workers discuss decomposition into multiplet bases [21] and recursion relations [22].

Typesetting

All birdtracks in these notes where drawn with `JaxoDraw` [23].

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Birdtracks for SU(N)

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Received 25-07-2017
Accepted 19-06-2018
Published 27-09-2018
doi:10.21468/SciPostPhysLectNotes.3