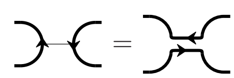
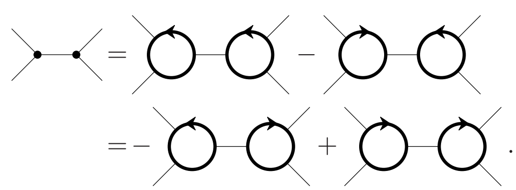
9.14 $SU(n)$, $U(n)$ EQUIVALENCE IN ADJOINT REP

The following simple observation speeds up evaluation of pure adjoint rep group-theoretic weights ($3n$-j)'s for $SU(n)$: The adjoint rep weights for $U(n)$ and $SU(n)$ are identical. This means that we can use the $U(n)$ adjoint projection operator

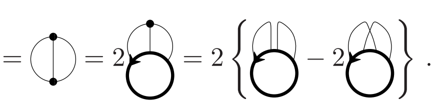
$U(n)$:  (9.118)

instead of the traceless $SU(n)$ projection operator (9.54), and halve the number of terms in the expansion of each adjoint line.

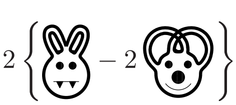
Proof: Any internal adjoint line connects two C_{ijk}'s:



The trace part of (9.54) cancels on each line; hence, it does not contribute to the pure adjoint rep diagrams. As an example, we reevaluate the adjoint quadratic casimir for $SU(n)$:

$C_A N =$ 

Now substitute the $U(n)$ adjoint projection operator (9.118):

$C_A N = 2$  $= 2n(n^2 - 1)$,

in agreement with the first exercise of section 2.2.