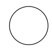
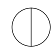
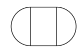
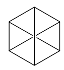
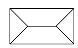
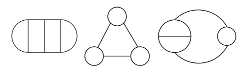
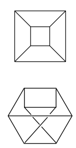
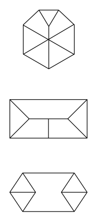
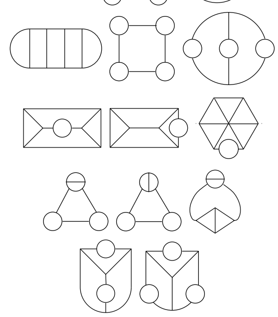
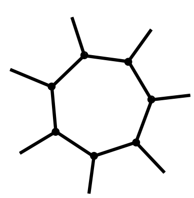
	Skeletons	Vertex insertions	Self-energy insertions	Total number
1-j				1
3j				1
6-j				2
9-j				5
12-j				16

Table 5.1 Topologically distinct types of Wigner $3n$-j coefficients, enumerated by drawing all possible graphs, eliminating the topologically equivalent ones by hand. Lines meeting in any 3-vertex correspond to any three irreducible representations with a nonvanishing Clebsch-Gordan coefficient, so in general these graphs cannot be reduced to simpler graphs by means of such as the Lie algebra (4.47) and Jacobi identity (4.48).

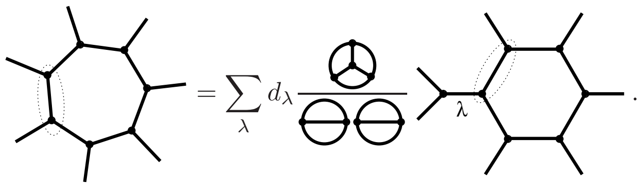
5.2 WIGNER $3n$-j COEFFICIENTS

An arbitrary higher-order contribution to a 2-particle scattering process will give a complicated matrix element. The corresponding energy levels, crosssections, *etc.*, are expressed in terms of scalars obtained by contracting all tensor indices; diagrammatically they look like “vacuum bubbles,” with $3n$ internal lines. The topologically distinct vacuum bubbles in low orders are given in table 5.1.

In group-theoretic literature, these diagrams are called $3n$-j symbols, and are studied in considerable detail. Fortunately, any $3n$-j symbol that contains as a subdiagram a loop with, let us say, seven vertices,

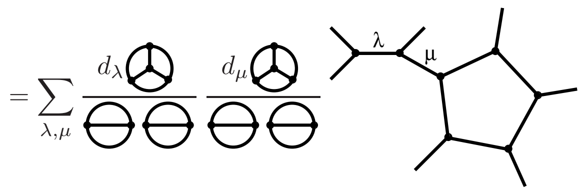
 ,

can be expressed in terms of 6-j coefficients. Replace the dotted pair of vertices by the cross-channel sum (5.13):



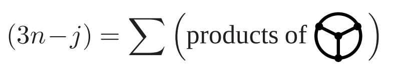
(5.16)

Now the loop has six vertices. Repeating the replacement for the next pair of vertices, we obtain a loop of length five:



(5.17)

Repeating this process we can eliminate the loop altogether, producing 5-vertex-trees times bunches of 6-j coefficients. In this way we have expressed the original $3n$-j coefficients in terms of $3(n$-1)-j coefficients and 6-j coefficients. Repeating the process for the $3(n$-1)-j coefficients, we eventually arrive at the result that



(5.18)