

The antisymmetrization tensor $A_{a_1 a_2 \dots, b_1 \dots b_p}$ has nonvanishing components, only if all lower (or upper) indices differ from each other. If the defining dimension is smaller than the number of indices, the tensor A has no nonvanishing components:

$$\begin{array}{c} \text{---} \\ 1 \\ \text{---} \\ 2 \\ \text{---} \\ \vdots \\ \text{---} \\ p \end{array} = 0 \quad \text{if } p > n. \quad (6.24)$$

This identity implies that for $p > n$, not all combinations of p Kronecker deltas are linearly independent. A typical relation is the $p = n + 1$ case

$$0 = \begin{array}{c} \text{---} \\ \text{---} \\ \vdots \\ \text{---} \\ 1 \quad 2 \quad \dots \quad n+1 \end{array} = \begin{array}{c} \text{---} \\ \text{---} \\ \vdots \\ \text{---} \\ 1 \quad 2 \quad \dots \quad n \end{array} - \begin{array}{c} \text{---} \\ \text{---} \\ \vdots \\ \text{---} \\ 1 \quad 2 \quad \dots \quad n \end{array} + \begin{array}{c} \text{---} \\ \text{---} \\ \vdots \\ \text{---} \\ 1 \quad 2 \quad \dots \quad n \end{array} - \dots \quad (6.25)$$

For example, for $n = 2$ we have

$$\begin{aligned} n = 2 : \quad 0 &= \begin{array}{c} f \quad e \quad d \\ \left| \quad \left| \quad \left| \right. \right. \\ a \quad b \quad c \end{array} - \begin{array}{c} \diagdown \quad \diagup \\ \diagup \quad \diagdown \end{array} - \begin{array}{c} \diagdown \quad \diagup \\ \diagdown \quad \diagup \end{array} + \begin{array}{c} \diagdown \quad \diagup \\ \diagdown \quad \diagup \end{array} + \begin{array}{c} \diagdown \quad \diagup \\ \diagdown \quad \diagup \end{array} - \begin{array}{c} \diagdown \quad \diagup \\ \diagdown \quad \diagup \end{array} \\ &= \delta_a^f \delta_b^e \delta_c^d - \delta_a^f \delta_c^e \delta_b^d - \delta_b^f \delta_a^e \delta_c^d + \delta_b^f \delta_c^e \delta_a^d + \delta_c^f \delta_a^e \delta_b^d - \delta_c^f \delta_b^e \delta_a^d. \end{aligned} \quad (6.26)$$