group theory - **week** 7

Lorenz to Van Gogh

Georgia Tech PHYS-7143

Homework HW7   due Thursday, October 12, 2017

== show all your work for maximum credit,
== put labels, title, legends on any graphs
== acknowledge study group member, if collective effort
== if you are LaTeXing, here is the source code

Exercise 7.1 *Product of two groups*   2 points
Exercise 7.2 *Space group*   2 points
Work through example 24.2 *Unrestricted symbolic dynamics*   6 points

Total of 12 points = 100 % score. Extra points accumulate, can help you later if you miss a few problems.

2017-10-03 Predrag Lecture 13 Fundamentalist vision

How I think of the fundamental domain is explained in my online lectures, Week 14, in particular the snippet Regular representation of permuting tiles. Unfortunately - if I had more time, that would have been shorter, this goes on and on, Week 15, lecture *29. Discrete symmetry factorization*, and by the time the dust settles, I do not have a gut feeling for the boundary conditions when it comes to higher-dimensional irreps (see also last week's sect. 6.1 *Discussion*). Kevin is working up an example for equilateral triangle wave functions...

2017-10-05 Predrag Lecture 14 Diffusion confusion

Read ChaosBook.org Chapter 24 *Deterministic diffusion* (last updated October 4, 2017). You also might find my online lectures, Week 13 helpful. I have also added ChaosBook.org Appendix A24 *Deterministic diffusion* (last updated October 4, 2017), but you probably do not need to read that.

7.1 Literature

The latest news, from Emory Math Department: A pariah is real! The simple finite groups fit into 18 families, except for the 26 sporadic groups. 20 sporadic groups AKA the Happy Family are parts of the Monster group. The remaining six loners are known as the pariahs. (Check the previous week notes sect. 5.1 *Literature* for links to the Ree group and the whole classification.)

Exercises

7.1. **Product of two groups.** Let G_1 and G_2 be two finite groups. The elements of the product set $G = G_1 \times G_2$ are defined as pairs (g_1, g_2), $g_1 \in G_1$ $g_2 \in G_2$.

    (a) Show that G is a group with the multiplication operation $(g_1, g_2) \cdot (g_1', g_2') = (g_1 g_1', g_2 g_2')$.

Let D_1 be an irreducible representation of G_1 and let D_2 be an irreducible representation of G_2. For each $g = (g_1, g_2) \in G$ define $D(g) = D_1(g_1) \times D_2(g_2)$

    (b) Show that $D = D_1 \times D_2$ is an irreducible representation of G. What are the characters of D?

7.2. **Space group.**

    (a) Show that for any space group, the translations by vectors from Bravais lattice form a normal subgroup.

    (b) Can rotations of the lattice at a fixed point constitute a normal subgroup of a space group?

<div align="right">(B. Gutkin)</div>