group theory - week 14

Flavor SU(3)

Georgia Tech PHYS-7143

Homework HW14 due Tuesday, November 28

== show all your work for maximum credit,
== put labels, title, legends on any graphs
== acknowledge study group member, if collective effort
== if you are LaTeXing, here is the source code

Exercise 14.1 *Gell-Mann–Okubo mass formula* 8 points

Bonus points
Exercise 15.3 *Young tableaux for SU*(3) 3 points
Exercise 15.4 *Irrep projection operators for unitary groups* 5 points

Total of 10 points = 100 % score. Extra points accumulate, can help you later if you miss a few problems.

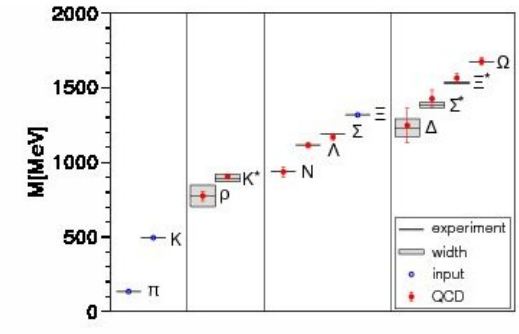

Figure 14.1: A lattice gauge theory calculation of the light QCD spectrum. Horizontal lines and bands are the experimental values with their decay widths. The π, K and and Ξ have no error bars because they are used to set the light and strange quark masses and the overall scale respectively. From Scholarpedia.

2017-11-21 Predrag Lecture 26 Flavor SU(3)

Gutkin notes, Lect. 11 *Strong interactions: flavor SU(3)*. Heisenberg isospin SU(2). Gell-Mann flavor SU(3). Gell-Mann-Okubo mass formula.

2017-11-24 Predrag (No lecture) Young tableaux

Young tableaux for SU(3) and SU(n) have not yet been covered in the lectures, but you can easily learn them yourself, from, for example, Gutkin notes, Lect. 12 *Young tableaux*. Boris Gutkin is a grownup, beyond learning new stuff, so he follows old fashioned references such as Fulton and Harris [4]. The resulting simple recipe with 0 explanation can be found, for example, here *C.G. Wohl*.

A modern exposition is given in *Group Theory – Birdtracks, Lie's, and Exceptional Groups,* Chapt. 9 *Unitary groups.* Currently I am a fan of the Alcock-Zeilinger algorithm [1–3], based on the simplification rules of ref. [2], which leads to explicitly Hermitian and compact expressions for the projection operators. Alcock-Zeilinger fully supersedes Cvitanović's formulation, and any future exposition of birdtracks reduction of SU(N) tensor products into irreducible representations will be based on the Alcock-Zeilinger algorithm.

The Gell-Mann-Okubo mass sum rules [5–7] are an easy consequence of the approximate SU(3) flavor symmetry. Determination of quark masses is much harder - they are parameters of the standard model, determined by optimizing the spectrum of particle masses obtained by lattice QCD calculations as compared to the experimental baryon and meson masses. The best determination of the mass spectrum as of 2012 is given in figure 14.1. Up, down quarks are about 3 and 6 MeV, respectively, with strange quark mass about 100 MeV, all with large error brackets. As of 2017, I have not found an update to figure 14.1, but the latest on the subject can probably be traced in Georg von Hippel's latticeqcd.blogspot.com.

References

[1] J. Alcock-Zeilinger and H. Weigert, "Transition operators", J. Math. Phys. **58**, 051702 (2016).

[2] J. Alcock-Zeilinger and H. Weigert, "Compact Hermitian Young projection operators", J. Math. Phys. **58**, 051702 (2017).

[3] J. Alcock-Zeilinger and H. Weigert, "Simplification rules for birdtrack operators", J. Math. Phys. **58**, 051701 (2017).

[4] W. Fulton and J. Harris, *Representation Theory* (Springer, New York, 1991).

[5] M. Gell-Mann, *The Eightfold Way: A Theory of Strong Interaction Symmetry*, Synchrotron Laboratory Report CTSL-20 (CalTech, 1961).

[6] M. Gell-Mann, "Symmetries of baryons and mesons", Phys. Rev. **125**, 1067–1084 (1962).

[7] S. Okubo, "Note on unitary symmetry in strong interactions", Progr. Theor. Phys. **27**, 949–966 (1962).

Exercises

14.1. **Gell-Mann–Okubo mass formula.** The mass symmetry-breaking interaction for an isospin multiplet is proportional to the 3rd component of the isospin operator, I_3. Similarly, the symmetry-breaking interaction of SU(3) for the meson octet is given by the 8th component of the octet operator $Y = \lambda_8$. Derive the GMO mass formula for mesons

$$m_\eta^2 = \frac{4\, m_K^2 - m_\pi^2}{3} \,. \tag{14.1}$$

by eliminating the parameter for the strength of this interaction, as in Gutkin lecture notes, Lect. 11 *Strong interactions: flavor SU*(3) .