group theory - week 7

QM applications

Georgia Tech PHYS-7143

Homework HW7 due Tuesday, March 1, 2016

== show all your work for maximum credit,
== put labels, title, legends on any graphs
== acknowledge study group member, if collective effort
== if you are LaTeXing, here is the source code

Exercise 7.1 *Perturbation of T_d symmetry* 6 points
Exercise 7.2 *Two particles in a potential* 4 points

Bonus points
Exercise 7.3 *Product of two groups* 2 points

Total of 12 points = 100 % score. Extra points accumulate, can help you later if you miss a few problems.

2016-02-23 Predrag Lecture 13 Fundamentalist vision

How I think of the fundamental domain is explained in my online lectures, Week 14, in particular the snippet Regular representation of permuting tiles. Unfortunately - if I had more time, that would have been shorter, this goes on and on, Week 15, lecture *29. Discrete symmetry factorization*, and by the time the dust settles, I do not have a gut feeling for the boundary conditions when it comes to higher-dimensional irreps...

2016-02-25 Boris Lecture 14 Quantum mechanics applications

Gutkin lecture notes Lecture 6 *Applications II. Quantum Mechanics*, Sect. *2. Perturbation theory*.

Exercises

7.1. **Perturbation of T_d symmetry.**
A non-relativistic charged particle moves in an infinite bound potential $V(x)$ with T_d symmetry. Consult exercise 5.1 *Vibration Modes of CH_4* for the character table and other T_d details.

(a) What are the degeneracies of the quantum energy levels? How often do they appear relative to each other (i.e., what is the level density)?

A weak constant electric field is now added now along one of the $2\pi/3$ rotation axes, splitting energy levels into multiplets.

(b) What is the symmetry group of the system now?

(c) How are the levels of the original system split? What are the new degeneracies?

7.2. **Two particles in a potential.**
Two distinguishable particles of the same mass move in a 2-dimensional potential $V(r)$ having D_4 symmetry. In addition they interact with each other with the term $\lambda W(|\boldsymbol{r}_1 - \boldsymbol{r}_2|)$.

(a) What is the symmetry group of the Hamiltonian if $\lambda = 0$? If $\lambda \neq 0$?.

(b) What are the degeneracies of the energy levels if $\lambda = 0$?

(c) Assuming that $\lambda \ll 1$ (weak interaction), describe the energy level structure, i.e., degeneracies and quasi-degeneracies of the energy levels. What will be the answer if the interaction is strong?

Hint: when interaction is weak we can think about it as perturbation.

7.3. **Product of two groups.** Let G_1 and G_2 be two finite groups. The elements of the product set $G = G_1 \times G_2$ are defined as pairs (g_1, g_2), $g_1 \in G_1$ $g_2 \in G_2$.

(a) Show that G is a group with the multiplication operation $(g_1, g_2) \cdot (g_1', g_2') = (g_1 g_1', g_2 g_2')$.

Let D_1 be an irreducible representation of G_1 and let D_2 be an irreducible representation of G_2. For each $g = (g_1, g_2) \in G$ define $D(g) = D_1(g_1) \times D_2(g_2)$

(b) Show that $D = D_1 \times D_2$ is an irreducible representation of G. What are the characters of D?

7.4. **Selection rules for T_d symmetry.**
The setup is the same as in exercise 7.1, but now assume that instead of a constant field, a time dependent electric field $\boldsymbol{E}_0 \cos(\omega t)$ is added to the system, with $\boldsymbol{E}_0$ not necessarily directed along any of the symmetry axes. In general, when $|E_n - E_m| = \hbar\omega$, such time-dependent perturbation induces transitions between energy levels E_n and E_m.

(a) What are the selection rules? Between which energy levels of the system are transitions possible?

(b) Would the answer be different if a magnetic field $\boldsymbol{B}_0 \cos(\omega t)$ is added instead? Explain how and why.