group theory - week 14

Birdtracks; Young tableaux

Georgia Tech PHYS-7143

Homework HW14 due Thursday, April 28

== show all your work for maximum credit,
== put labels, title, legends on any graphs
== acknowledge study group member, if collective effort
== if you are LaTeXing, here is the source code

Exercise 14.1 *Representations of* $SU(3)$	5 points
Exercise 14.2 *Young tableaux for* S_5	3 points
Exercise 14.4 *Lie algebra from invariance*	2 points

Bonus points

Exercise 14.3 *Young tableaux for* $SU(3)$	3 points
Exercise 14.5 *Irrep projection operators for unitary groups*	5 points

Total of 10 points = 100 % score. Extra points accumulate, can help you later if you miss a few problems.

2016-04-19 Predrag Lecture 27 Birdtracks

Predrag's monograph [1], *Group Theory Birdtracks, Lie's, and Exceptional Groups*, Sect. 2.1 *Basic concepts*, Sect. 2.1 *First example: $SU(n)$*. Chap. 1 explains pretty well what the monograph is about.

2016-04-21 Boris Lecture 28 Many particle systems. Young tableaux

Gutkin notes, Lect. 12 *Many particle systems. Young tableaux*.

Excerpt from Predrag's monograph [1], fetch it here: Sect. 9.3 *Young tableaux*.

References

[1] P. Cvitanović, *Group Theory - Birdtracks, Lie's, and Exceptional Groups* (Princeton Univ. Press, Princeton, NJ, 2008).

Exercises

14.1. **Representations of $SU(3)$.** Any irrep of $SU(3)$ can be labeled $D(p,q)$ by its highest weight $\lambda = p\lambda_1 + q\lambda_2$, where $\lambda_{1,2}$ are the two fundamental weights.

(a) Find all irreps $D(p,q)$ of $SU(3)$ with the dimensions less then 20 (see lecture notes for the dimensions of $D(p,q)$).

(b) Draw the lattice Λ generated by $\lambda_{1,2}$ and mark there all the weights v (i.e., lattice nodes) which belong to irrep. $D(3,0)$. Is $D(3,0)$ a real irrep?

(c) Consider product (reducible) representation $3 \otimes 3$, where $3 = D(1,0)$ is the fundamental irrep. Mark all the weights v on Λ which belong to $3 \otimes 3$. Using this find out decomposition of $3 \otimes 3$ into irreps:

$$3 \otimes 3 = \Box \oplus \triangle, \qquad \Box = ?, \qquad \triangle = ?$$

Hint: see lecture notes for similar exercise on $3 \otimes \bar{3}$.

(d) Using previous results find decomposition of $3 \otimes 3 \otimes 3$ into irreps.

(B. Gutkin)

14.2. **Young tableaux for S_5.**

(a) Draw all Young diagrams for the symmetric group S_5. How many irreducible representations has it? Which of the diagrams correspond to one-dimensional irreps?

(b) Find Young diagram corresponding to the irrep of S_5 with the largest dimension? Draw Young tableaux corresponding to this irrep/Young diagram. What is the dimension of this irrep?

(c) What are the dimensions of the remaining irreps?

(B. Gutkin)

14.3. **Young tableaux for $SU(3)$.** Solve exercise 14.1 (c,d) by using Young tableaux.

Remark: If Young tableaux for $SU(3)$ are not covered in the lectures, learn them yourself from, for example, *Group Theory Birdtracks, Lie's, and Exceptional Groups.* The resulting simple recipe with 0 explanation can be found, for example, here *C.G. Wohl*.

(B. Gutkin)

14.4. **Lie algebra from invariance.** Derive the Lie algebra commutator and the Jacobi identity as particular examples of the invariance condition, using both index and birdtracks notations. The invariant tensors in question are "the laws of motion," i.e., the generators of infinitesimal group transformations in the defining and the adjoint representations.

14.5. **Irrep projection operators for unitary groups.** Derive projection operators and dimensions for irreps of the Kronecker product of the defining and the adjoint reps of SU(n) listed in *Group Theory Birdtracks, Lie's, and Exceptional Groups*, Table 9.3. (Ignore "indices," we have not defined them.)