group theory - week 14

Flavor SU(3)

Georgia Tech PHYS-7143

Homework HW14 due Wednesday 2021-08-04

== show all your work for maximum credit,
== put labels, title, legends on any graphs
== acknowledge study group member, if collective effort
== if you are LaTeXing, here is the source code

Bonus points

Exercise 14.1 *Gell-Mann–Okubo mass formula*	8 points
Exercise 15.3 *Young tableaux for* $SU(3)$	3 points
Exercise 15.4 *Irrep projection operators for unitary groups*	5 points

All points are bonus points. Extra points accumulate, can help you if you had missed a few problems.

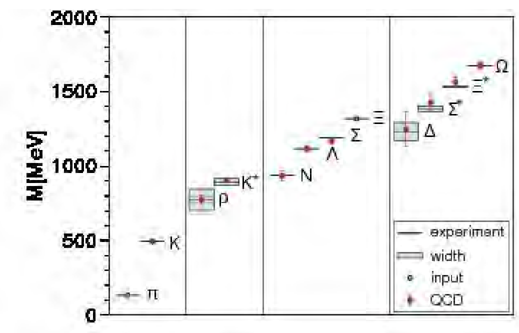

Figure 14.1: A lattice gauge theory calculation of the light QCD spectrum. Horizontal lines and bands are the experimental values with their decay widths. The π, K and and Ξ have no error bars because they are used to set the light and strange quark masses and the overall scale respectively. From Scholarpedia.

2021-07-22 Predrag Lecture 27 Flavor SU(3)

(Unedited 1:26:11 h) *Gell-Mann–Okubo mass formula.* Pions and kaons fit into an SU(3) octet, but the strange mesons masses are much larger, breaking the symmetry $\mathrm{SU}(3) \to \mathrm{SU}(2) \times U(1)$. That leads to a Gell-Mann – Okubo parameter-free constraints on various masses, which were verified to high accuracy, and lead to predictions of masses for yet undiscovered particles.

Gutkin notes, Lect. 11 *Strong interactions: flavor SU(3).* Heisenberg isospin SU(2). Gell-Mann flavor SU(3). Gell-Mann-Okubo mass formula.

The Gell-Mann-Okubo mass sum rules [1, 2, 4] are an easy consequence of the approximate SU(3) flavor symmetry. Determination of quark masses is much harder - they are parameters of the standard model, determined by optimizing the spectrum of particle masses obtained by lattice QCD calculations as compared to the experimental baryon and meson masses. The best determination of the mass spectrum as of 2012 is given in figure 14.1. Up, down quarks are about 3 and 6 MeV, respectively, with strange quark mass about 100 MeV, all with large error brackets. As of 2021, I have not found an update to figure 14.1, but the latest on the subject can probably be traced in Georg von Hippel's latticeqcd.blogspot.com.

14.1 Isotropic quantum harmonic oscillator

One of the "hidden" quantum symmetries is the SU(3) of the 3D isotropic quantum harmonic oscillator. Murgan and Zender [3] *Energy eigenvalues of the three-dimensional quantum harmonic oscillator from SU(3) cubic Casimir operator* is a nice pedagogical intro to this SU(3). Would prefer no explicit irreps (see www.birdtracks.eu) but working this out is a good (but long) exercise.

References

[1] M. Gell-Mann, *The Eightfold Way: A Theory of Strong Interaction Symmetry* (CalTech, 1961).

[2] M. Gell-Mann, "Symmetries of baryons and mesons", Phys. Rev. **125**, 1067–1084 (1962).

[3] R. Murgan and A. Zender, "Energy eigenvalues of the three-dimensional quantum harmonic oscillator from SU(3) cubic Casimir operator", Eur. J. Phys. **40**, 015405 (2018).

[4] S. Okubo, "Note on unitary symmetry in strong interactions", Progr. Theor. Phys. **27**, 949–966 (1962).

Exercises

14.1. **Gell-Mann–Okubo mass formula.** The mass symmetry-breaking interaction for an isospin multiplet is proportional to the 3rd component of the isospin operator, I_3. Similarly, the symmetry-breaking interaction of SU(3) for the meson octet is given by the 8th component of the octet operator $Y = \lambda_8$. Derive the GMO mass formula for mesons

$$m_\eta^2 = \frac{4\, m_K^2 - m_\pi^2}{3}\,. \tag{14.1}$$

by eliminating the parameter for the strength of this interaction, as in Gutkin lecture notes, Lect. 11 *Strong interactions: flavor SU*(3) .