Chapter 10. Qualitative dynamics, for pedestrians

Solution 10.1: Binary symbolic dynamics. *Read the text.*

Solution 10.2: Generating prime cycles. *(No solution available.)*

Solution 10.3: A contracting baker's map. *(No solution available.)*

Solution 10.4: Unimodal map symbolic dynamics. *Hint: write down an arbitrary binary number such as $\gamma = .1101001101000\ldots$ and generate the future itinerary S^+ by checking whether $f^n(\gamma)$ is greater or less than 1/2. Then verify that (??) recovers γ.*

Solution 10.5: Unimodal map kneading value. *(No solution available.)*

Solution 10.6: One-dimensional repellers. *(No solution available.)* ⇓PRELIMINARY

Solution 10.7: "Golden mean" pruned map. ⇑PRELIMINARY
(a) Consider the 3-cycle drawn in the figure. Denote the lengths of the two horizontal intervals by a and b. We have

$$\frac{a}{b} = \frac{b}{a+b},$$

so the slope is given by the golden mean, $\Lambda = \frac{b}{a} = \frac{1+\sqrt{5}}{2}$, and the piece-wise linear map is given by

$$f(x) = \begin{cases} \Lambda x\,, \ x \in [0, 1/2] \\ \Lambda(1-x)\,, \ x \in [1/2, 1] \end{cases}$$

(b) Evaluate

$$f\left(\frac{1}{2}\right) = \frac{1+\sqrt{5}}{4}, \quad f\left(\frac{1+\sqrt{5}}{4}\right) = \frac{-1+\sqrt{5}}{4}, \quad f\left(\frac{-1+\sqrt{5}}{4}\right) = \frac{1}{2}.$$

Once a point enters the region covered by the interval $\mathcal{M}$ of length $a+b$, bracketed by the 3-cycle, it will be trapped there forever. Outside $\mathcal{M}$, all points on unit interval will be mapped to $(0, 1/2]$, except for 0. The points in the interval $(0, \frac{-1+\sqrt{5}}{4})$ approach $\mathcal{M}$ monotonically.

(c) It will be in $(\frac{1}{2}, \frac{1+\sqrt{5}}{4})$.

(d) From (b), we know that except for the origin 0, all periodic orbits should be in $\mathcal{M}$. By (c), we cannot have the substring 00 in a periodic orbit (except for the fixed point at 0). Hence 00 is the only pruning block, *and the symbolic dynamics is a finite subshift, with alphabet $\{0, 1\}$ and only one grammar rule: a consecutive repeat of symbol 0 is inadmissible.*

(e) Yes. 0 is a periodic orbit with the symbol sequence $\overline{0}$. It is a repeller and no point in its neighborhood will return. So it plays no role in the asymptotic dynamics.

(Yueheng Lan)

Solution 10.8: Binary 3-step transition matrix. *(No solution available.)*

Solution 10.9: Heavy pruning. *(No solution available.)*