PHYS 7147

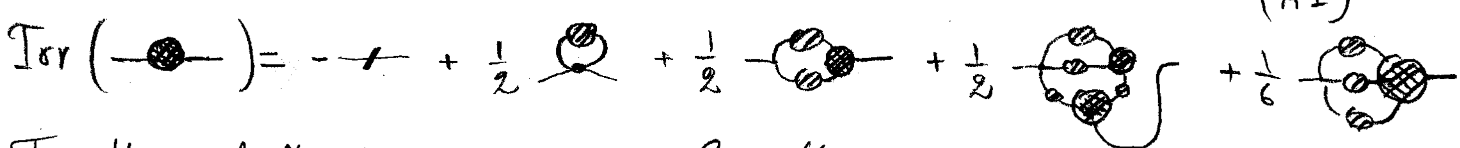
Problem set #3 Solutions

Rytis Paskauskas, 2003

2.I.3

I'm going to use shorthand notation "Irr(Diagram)" which will stand for taking the irreducible part of the diagram. For example, here's the Irr(2.35):

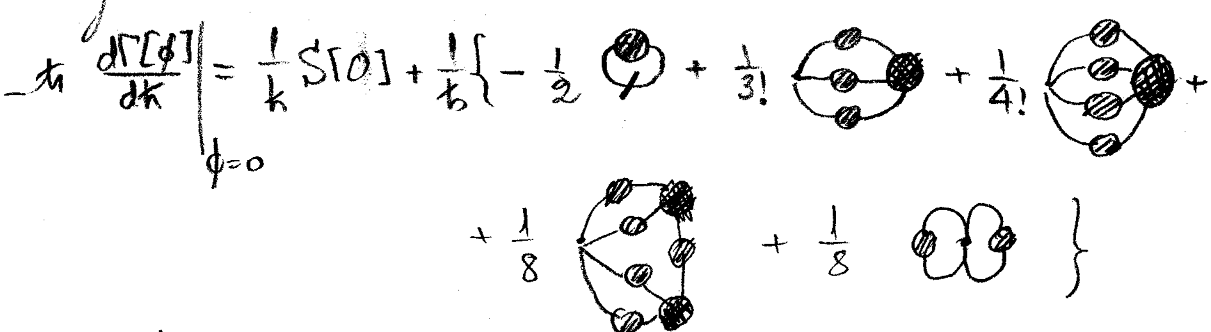
(A1)



In the following expressions I will only draw diagrams containing up to 3 loops (as asked by the problem).

$$\frac{1}{\hbar}\Gamma[0] = \left(\frac{1}{\hbar}\int_0^{\hbar} d\hbar \, \frac{d\Gamma[\phi]}{d\hbar}\right)\Bigg|_{\phi=0}$$

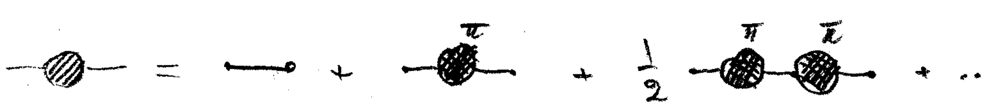
According to (2.45):



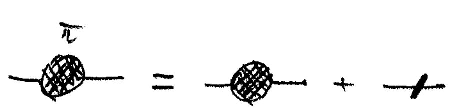
(A2)

Since $S[\phi] = -\frac{1}{2}\Delta^{-1}_{ij}\phi_i\phi_j + \frac{1}{3!}\gamma_{ijk}\phi_i\phi_j\phi_k + \dots$

$$S[0] = \emptyset .$$



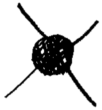
(eqn. before 2.32)

$\pi_{ij} = \Gamma_{ij} + \Delta^{-1}_{ij} \to$ 

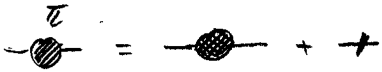
Since in (A2):

 comes with diagram with 1 loop, I will expand it to 2 loops

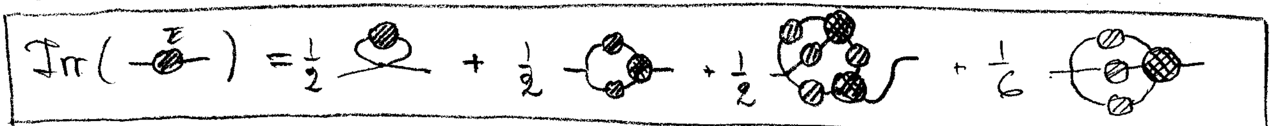
 comes with diag. with 2 loops, I'll expand it to 1 loop.

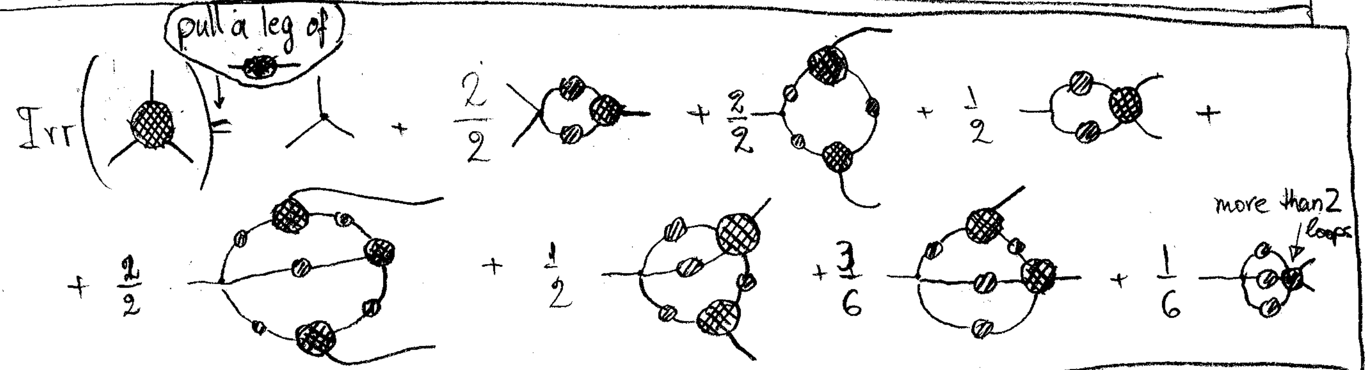
 comes with diag. with 3 loops I'll expand it to zero loops.

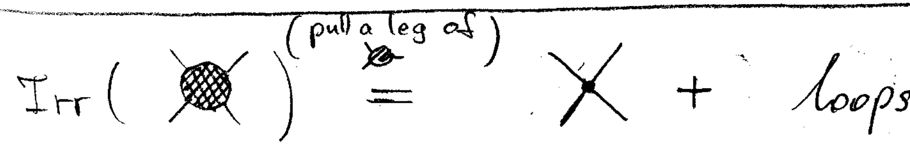
{  contains at least 1 loop, so it will be omitted }



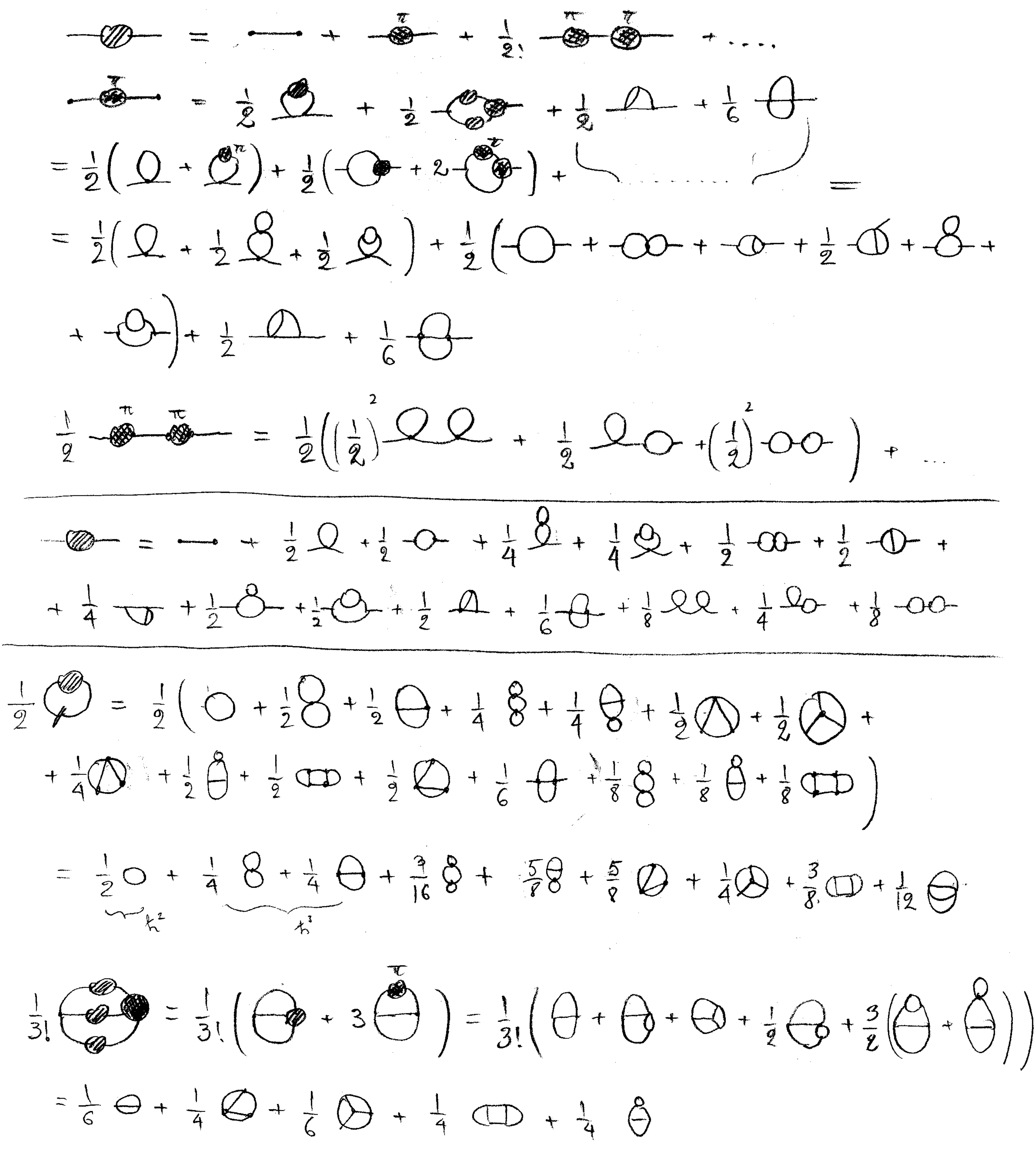
is done in the book (eqn 2.35)

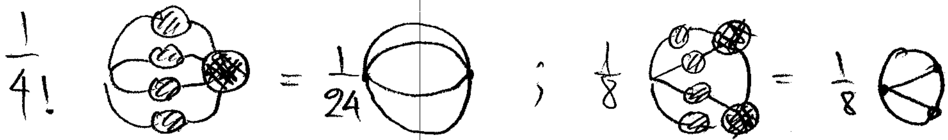


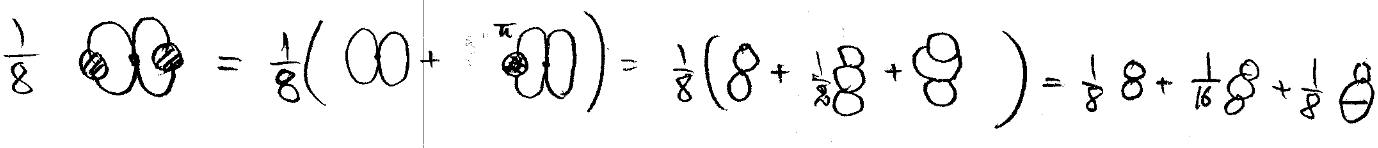


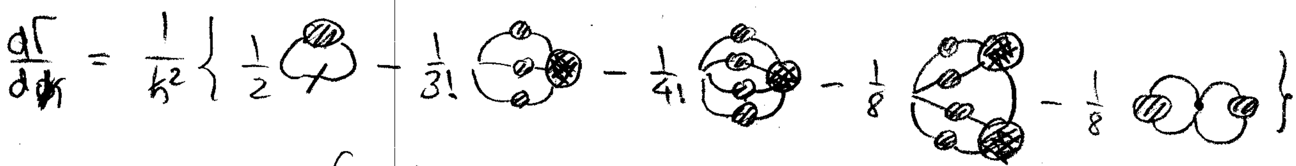


Up to 2 loops:









Coefficients in front of diagrams:

$\frac{1}{2}$ $\Big\} \times \hbar$

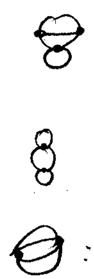
$\frac{1}{4} - \frac{1}{8} = \frac{1}{8}$

$\frac{1}{4} - \frac{1}{6} = \frac{1}{12}$ $\Big\} \times \hbar^2$

$\frac{1}{4} - \frac{1}{6} = \frac{1}{12}$

$\frac{3}{8} - \frac{1}{4} = \frac{1}{8}$

$\frac{5}{8} - \frac{1}{4} - \frac{1}{8} = \frac{1}{4}$



$\frac{5}{8} - \frac{1}{4} - \frac{1}{8} = \frac{1}{4}$

$\frac{3}{16} - \frac{1}{16} = \frac{1}{8}$

$\frac{1}{12} - \frac{1}{24} = \frac{1}{24}$ $\Big\} \hbar^3$