(e) The stretch is

$$s + vt - x = rs + \frac{v}{\omega}\sin\omega t + (1-r)s\cos\omega t$$
$$= rs + (1-r)s\frac{\cos(\omega t - \alpha)}{\cos\alpha}$$

The minimum stretch is $s(r - (1-r)/\cos\alpha)$ for $\omega t = \pi + \alpha$. For the minimum stretch to be positive, one must require that $r > (1-r)/\cos\alpha$, or

$$\frac{v^2}{s^2\omega^2} < (1-r)(2r-1) \ .$$

This inequality can only be fulfilled for $r > 1/2$.

(f) When the block stops the stretch is $s_0 = s(r - (1-r)\cos 3\alpha/\cos\alpha)$. The stretch grows to $s_0 + v\Delta t$ a time Δt after the body stops. The body starts to move again when the stretch is s, or $v\Delta t = s - s_0$. If this is negative, the body never stops.

9.8 Writing out the 6 terms of the determinant, the characteristic equation becomes $\det[\boldsymbol{\sigma} - \lambda\mathbf{1}] = -\lambda^3 + I_1\lambda^2 - I_2\lambda + I_3$. An asymmetric stress tensor also has the invariant $I_4 = \sum_{ij}\sigma_{ij}(\sigma_{ij} - \sigma_{ji})$ which vanishes for a symmetric stress tensor.

10 Strain

10.3 We must solve

$$\nabla_x u_x = \nabla_y u_y = \nabla_z u_z = 0$$
$$\nabla_y u_z + \nabla_z u_y = \nabla_z u_x + \nabla_x u_z = \nabla_x u_y + \nabla_y u_x = 0$$

From the first we get that u_x can only depend on y and z, and for the second derivatives we get

$$\nabla_y^2 u_x = -\nabla_y\nabla_x u_y = -\nabla_x\nabla_y u_y = 0$$
$$\nabla_z^2 u_x = -\nabla_z\nabla_x u_z = -\nabla_x\nabla_z u_z = 0$$
$$\nabla_y\nabla_z u_x = -\nabla_y\nabla_x u_z = -\nabla_x\nabla_y u_z = \nabla_x\nabla_z u_y = \nabla_z\nabla_x u_y = -\nabla_z\nabla_y u_x$$

From the last equation we get $\nabla_y\nabla_z u_x = 0$. Consequently, we must have $u_x = A + Dy + Ez$ and similar results for u_y and u_z. The vanishing of the shear strains relates some of the constants.

10.4 The strain gradients become

$$\{\nabla_j u_i\} = \alpha\begin{pmatrix} 0 & 2y & 0 \\ y & x & 0 \\ 0 & 0 & 0 \end{pmatrix} \ , \qquad \text{(10-A1)}$$

and are small for $|\alpha| \ll 1/L$. Cauchy's strain tensor becomes in the same approximation

$$\{u_{ij}\} = \alpha\begin{pmatrix} 0 & \frac{3}{2}y & 0 \\ \frac{3}{2}y & x & 0 \\ 0 & 0 & 0 \end{pmatrix} \ , \qquad \text{(10-A2)}$$

10.5 The eigenvalue equation becomes $\lambda^2 - x\lambda - \frac{9}{4}y^2 = 0$, and has the solution $\lambda = \frac{1}{2}(x \pm \sqrt{x^2+9y^2})$. The corresponding (unnormalized) eigenvectors are $(3y, x \pm \sqrt{x^2+9y^2}, 0)$ and $(0,0,1)$.

10.6 Successive needle transformations

$$a''_i = \sum_j (\delta_{ij} + \nabla'_j u'_i) a'_j = \sum_{jk} (\delta_{ij} + \nabla'_j u'_i)(\delta_{jk} + \nabla_k u_j) a_k \ .$$

Then

$$\delta_{km} + 2\widetilde{u}_{km} = \sum_{ijl} (\delta_{ij} + \nabla'_j u'_i)(\delta_{jk} + \nabla_k u_j)(\delta_{il} + \nabla'_l u'_i)(\delta_{lm} + \nabla_m u_l) \ ,$$

and finally

$$\widetilde{u}_{km} = u_{km} + \sum_{jl} u'_{jl} (\delta_{jk} + \nabla_k u_j)(\delta_{lm} + \nabla_m u_l) \ .$$

10.10 $\kappa^3 - 1$.

10.12

$$\{u_{ij}\} = \frac{1}{2} \begin{pmatrix} 2A + A^2 + C^2 & (A-B+2)C & 0 \\ (A-B+2)C & 2B + C^2 + B^2 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

10.13 Use the general transformation (10-34) and write the condition for vanishing strain as

$$\sum_k F_{ik} F_{jk} = \sum_k F_{ki} F_{kj} = \delta_{ij} \qquad \text{(10-A3)}$$

which shows that $\mathbf{F}$ is everywhere an orthogonal matrix. Differentiating after x_l we get

$$\sum_k \frac{\partial F_{ik}}{\partial x_l} F_{jk} + \sum_k F_{ik} \frac{\partial F_{jk}}{\partial x_l} = 0 \qquad \text{(10-A4)}$$

or after multiplying with F_{jm} and summing

$$\frac{\partial F_{im}}{\partial x_l} = -\sum_{jk} F_{ik} F_{jm} \frac{\partial F_{jk}}{\partial x_l} \qquad \text{(10-A5)}$$

Now we use that

$$\frac{\partial F_{jk}}{\partial x_l} = \frac{\partial^2 x'_j}{\partial x_k \partial x_l} = \frac{\partial F_{jl}}{\partial x_k} \qquad \text{(10-A6)}$$

and by repeated applications of the rules we find

$$\begin{aligned} \frac{\partial F_{im}}{\partial x_l} &= -\sum_{jk} F_{ik} F_{jm} \frac{\partial F_{jl}}{\partial x_k} = \sum_{jk} F_{ik} F_{jl} \frac{\partial F_{jm}}{\partial x_k} \\ &= \sum_{jk} F_{ik} F_{jl} \frac{\partial F_{jk}}{\partial x_m} = -\sum_{jk} F_{ik} F_{jk} \frac{\partial F_{jl}}{\partial x_m} \\ &= -\frac{\partial F_{il}}{\partial x_m} = -\frac{\partial F_{im}}{\partial x_l} \end{aligned}$$

