Consequently, all the derivatives of F_{im} must vanish, so that it is a constant orthogonal matrix, *i.e.* a rotation (excluding reflections).

10.16

$$\{u_{ij}\} = \frac{1}{2}\begin{pmatrix} 34 & -9 & 0 \\ -9 & 73 & 17 \\ 0 & 17 & 29 \end{pmatrix}$$

10.17 Use that the determinant of a product of matrices is the product of the determinant to get $\det \delta_{ij} + 2u_{ij} = \det |\delta_{ij} + \nabla_j u_i|^2$.

11 Elasticity

11.1 Expanding to first order in $r - a$, we have for r close to a

$$f(r) = f(a) + (r-a)f'(a) \ .$$

With $\Delta F = f(r) - f(a)$, $\Delta L = r - a$, and $k = f'(a)$ this is Hooke's law.

11.2 b) $\omega^2 = 4\sin^2 \frac{k}{2}$.

11.6 The absolute minimum of the coefficient of the first term happens for $\alpha = 1/3$.

11.7

(a) Follows from the symmetry of σ_{ij} and u_{ij}.

(b) In order for $\delta u_{ij} \Delta\sigma_{ij} = \delta u_{ij}\lambda_{ijkl}u_{kl}$ to become a total differential $\delta(\frac{1}{2}\lambda_{ijkl}u_{ij}u_{kl})$, the 9×9-matrix $\lambda_{(ij)(kl)}$ must be symmetric.

12 Solids at rest

12.4 We use that u_{zz} is linear in x and y, of the form

$$u_{zz} = \nabla_z u_z = \alpha - \beta_x x - \beta_y y$$

and consequently

$$u_z = a_z - \phi_y x + \phi_x y + \alpha z - \beta_x xz - \beta_y yz$$

where the coefficients are all constants. Using this result, we find $\nabla_x u_x = \nabla_y u_y = -\nu(\alpha - \beta_x x - \beta_y y)$. Integrating and demanding that $u_{xy} = u_{xz} = u_{zy} = 0$, we obtain the most general form

$$u_x = a_x - \phi_z y + \phi_y z - \alpha\nu x + \frac{1}{2}\beta_x \left(z^2 - \nu(x^2 - y^2)\right) - \beta_y \nu xy \ , \qquad \text{(12-A1a)}$$

$$u_y = a_y + \phi_z x - \phi_x z - \alpha\nu y + \frac{1}{2}\beta_y \left(z^2 - \nu(y^2 - x^2)\right) - \beta_x \nu xy \ , \qquad \text{(12-A1b)}$$

$$u_z = a_z - \phi_y x + \phi_x y + \alpha z - \beta_x xz - \beta_y yz \ . \qquad \text{(12-A1c)}$$

Here (a_x, a_y, a_z) represents simple translations of the body, and (ϕ_x, ϕ_y, ϕ_z) simple rotations around the coordinate axes. The coefficient α corresponds to a uniform stretching, and only (β_x, β_y) represents bending into the coordinate directions.

12.5 Same general solution as for the pressurized tube but with a and b interchanged

$$A = -\frac{1}{2(\lambda+\mu)}\frac{b^2}{b^2-a^2}P = -(1+\sigma)(1-2\sigma)\frac{b^2}{b^2-a^2}\frac{P}{E}\ ,$$
$$B = -\frac{1}{2\mu}\frac{a^2b^2}{b^2-a^2}P = -(1+\sigma)\frac{a^2b^2}{b^2-a^2}\frac{P}{E}\ .$$

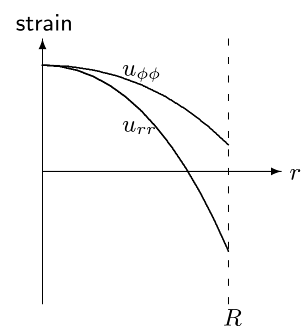
The rest is straightforward.

12.6 a) The centrifugal force density is radial and given by $f_r = \rho_0\Omega^2 r$. b) The general solution to (12-46) is

$$u_r = Ar + \frac{B}{r} - \frac{1}{8}\frac{\rho_0\Omega^2}{\lambda+2\mu}\, r^3\ . \qquad (12\text{-A2})$$

where A and B are integration constants. Use that u_r must be finite for $r = 0$ and $\sigma_{rr} = 0$ for $r = a$. The final solution becomes

$$u_r = \frac{1}{8}\frac{\rho_0\Omega^2 a^2}{\lambda+2\mu} r\left(3-2\nu-\frac{r^2}{a^2}\right)\ , \qquad (12\text{-A3})$$



Sketch of strains in the massive rotating cylinder.

c) The strains are

$$u_{rr} = \frac{1}{8}\frac{\rho_0\Omega^2 a^2}{\lambda+2\mu}\left(3-2\nu-3\frac{r^2}{a^2}\right)\ , \quad u_{\phi\phi} = \frac{1}{8}\frac{\rho_0\Omega^2 a^2}{\lambda+2\mu}\left(3-2\nu-\frac{r^2}{a^2}\right)\ . \qquad (12\text{-A4})$$

The radial strain is positive for $r = 0$, vanishes for $r = a\sqrt{1-2\nu/3}$, and is negative for $r = a$. d) Breakdown happens for $r = 0$ where the extension and tension is maximal.

12.9 The total force is

$$\mathcal{F}_z = \int_A \sigma_{zz}\, ds_z = -\frac{E}{R}I_x\ . \qquad (12\text{-A5})$$

If the area A is symmetric under reflection in the y-axis, then $I_x = 0$.

12.10 Letting $x \to x + \alpha$ in (12-20), we get

$$\begin{aligned} u_x &= \frac{\nu}{2R}\alpha^2 + \frac{\nu}{R}\alpha x + \frac{1}{2R}(z^2 + \nu(x^2 - y^2))\ , \\ u_y &= \frac{\nu}{R}\alpha y + \frac{\nu}{R}xy\ , \\ u_z &= -\frac{1}{R}\alpha z - \frac{1}{R}xz\ . \end{aligned}$$

The first column represents a simple translation and the second a uniform stretching of the form (11-24).

13 Vibrations

13.1 For an arbitrary vector field we first find a solution ψ to the equation

$$\boldsymbol{\nabla}^2\psi = \boldsymbol{\nabla}\cdot\boldsymbol{u} \qquad (13\text{-A1})$$