Remark 1.1. Projection operators. The construction of projection operators given in sect. 1.5.1 is taken from refs. [3, 4]. Who wrote this down first we do not know, lineage certainly goes all the way back to Lagrange polynomials [10], but projection operators tend to get drowned in sea of algebraic details. Arfken and Weber [1] ascribe spectral decomposition (1.36) to Sylvester. Halmos [6] is a good early reference - but we like Harter's exposition [7–9] best, for its multitude of specific examples and physical illustrations. In particular, by the time we get to (1.33) we have tacitly assumed full diagonalizability of matrix $\mathbf{M}$. That is the case for the compact groups we will study here (they are all subgroups of $\mathrm{U}(n)$) but not necessarily in other applications. A bit of what happens then (nilpotent blocks) is touched upon in example 1.4. Harter in his lecture Harter's lecture 5 (starts about min. 31 into the lecture) explains this in great detail - its well worth your time.

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